

Application of Modified Compression Field Theory for Shear Design in the AASHTO LRFD Code

There are benefits and drawbacks to using the new AASHTO bridge design method to calculate the shear capacity of concrete instead of the traditional design method.

JASON VARNEY

Over the years, engineers have performed a great deal of research and experimentation in an attempt to accurately qualitatively and quantitatively describe the behavior and failure mechanisms of concrete in shear. Unlike materials such as steel, the non-homogeneity and inelasticity of concrete as a building material makes this behavior very difficult to quantify, and mod-

ern concrete design methods are continually being revised and reworked in order to better represent the true behavior of the material when subjected to shear forces. The American Association of State Highway and Transportation Officials (AASHTO) load and resistance factor design (LRFD) bridge design code has incorporated a new approach for analyzing and designing for shear in concrete bridges (issued as *AASHTO LRFD Bridge Design Specifications, Fourth Edition*¹). This new approach is based on the modified compression field theory (MCFT).

Although the exact behavior of concrete will likely never be fully resolved, newer, more involved theories (such as MCFT) are being adopted within the engineering world in order to more accurately model the true shear strength of concrete. It remains to be seen whether these emerging theories provide more advantages in the economics of concrete design than disadvantages in their calculation.

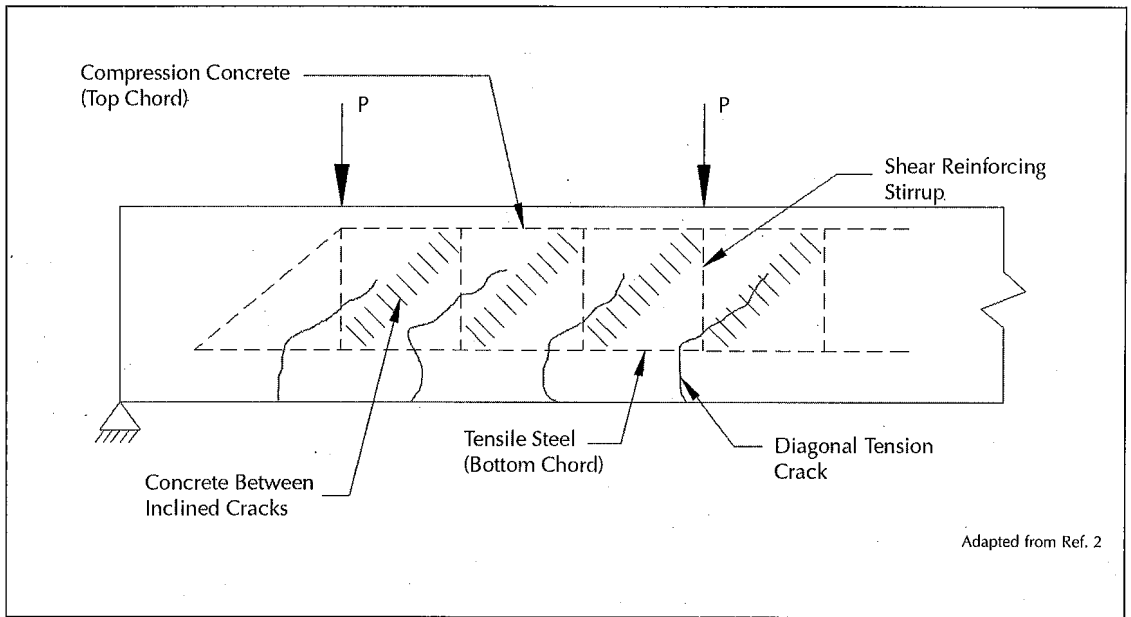


FIGURE 1. Truss model of axial shear forces in a reinforced concrete beam.

Traditional Method Derivation

Before adoption of the new MCFT approach to the AASHTO LRFD bridge design code, engineers used the “traditional” method to design reinforced concrete bridges for shear. This traditional approach to concrete shear analysis incorporates the assumption that the nominal shear capacity of a reinforced concrete section, V_n , is composed of two components:

- the resistance provided by the concrete, V_c ; and,
- the resistance provided by the steel reinforcement stirrups, V_s .

The contributing forces can be summed up with a simplified truss model in which axial shear is resolved into compression and tension struts, with the concrete taking compression and the steel stirrups taking tension. This truss analogy can be seen in Figure 1, along with a schematic of beam components and dimensions in Figure 2.

The variables in Figure 2 are defined as follows:

- s = spacing of shear stirrups
- s_1 = crack length

- α = stirrup angle
- β = crack angle
- C = compressive force resultant
- T = tensile force resultant
- d = distance from top face to tensile steel

From the given geometry, it can be determined that:

$$V_s = T_s \sin(\alpha) \tag{1}$$

where:

- T_s = the force resultant of all web stirrups across the diagonal crack, and
- α = the angle of the shear stirrups relative to the horizontal.

If n is equal to the number of stirrup spacings within crack length s_1 , then:

$$s_1 = n \cdot s = j d^* [\cot(\alpha) + \cot(\beta)] \tag{2}$$

where:

- β = the angle of the shear crack.

Dividing the stirrup force by the crack length, T_s/s_1 , and approximating j equal to 1, substitutions to Equation 2 can be made in order to obtain:

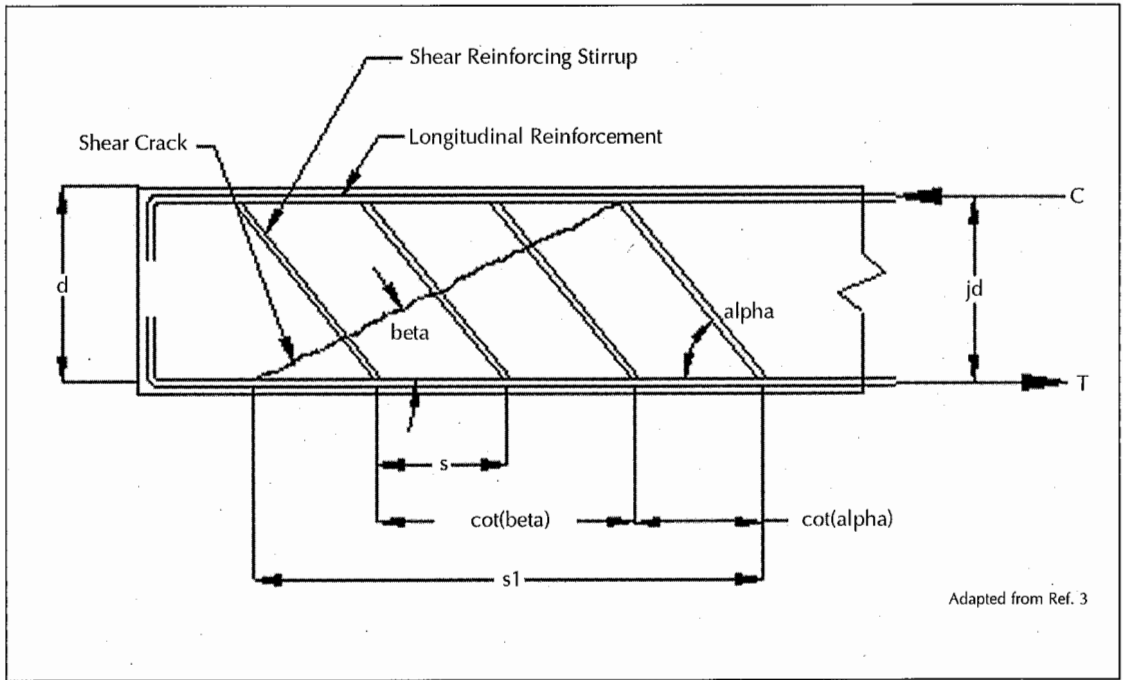


FIGURE 2. Schematic of a cracked shear-reinforced concrete beam.

$$(V_s/\sin(\alpha))/(d*(\cot(\alpha)+\cot(\beta))) \quad (3)$$

Given n stirrups over crack length s_1 , the total force resisted by the stirrups must equal T_s , or:

$$T_s = n * A_v * f_y \quad (4)$$

where:

A_v = the area of a single shear stirrup, and
 f_y = the yield strength of the steel.

With these relationships, the following can be inferred:

$$n * A_v = T_s / f_y = (V_s * n * s / \sin(\alpha)) / (d * (\cot(\alpha) + \cot(\beta)) * f_y) \quad (5)$$

By rearranging Equation 5, it can be restated as:

$$V_s = (A_v * f_y * d / s) * (\cot(\alpha) + \cot(\beta)) \quad (6)$$

The traditional model makes two simplifying assumptions as shown in Figure 3. First, the traditional approach assumes that shear cracks form at an angle, β , of 45 degrees to horizontal. Second, the stirrups

are assumed to be vertical; therefore, α is equal to 90 degrees. If β is equal to 45 degrees:

$$\begin{aligned} V_s &= (A_v * f_y * d / s) * (\sin(\alpha)) * (1 + \cot(\alpha)) \\ &= A_v * f_y * d / s * (\sin(\alpha) + \cos(\alpha)) \end{aligned} \quad (7)$$

Equation 7 can be simplified further by setting α to 90 degrees, and

$$\begin{aligned} V_s &= (A_v * f_y * d / s) \\ s &= (A_v * f_y * d) / V_s = (A_v * f_y * d) / (V_u / \phi - V_c) \end{aligned} \quad (9)$$

where:

V_u / ϕ = the required resistance; and,
 $V_c = (2\sqrt{f'_c}) * b_w * d$ (concrete shear strength determined by experimentation).

Using $(2\sqrt{f'_c}) * b_w * d$ to determine concrete shear strength, it is possible to determine if a concrete beam or slab is strong enough to resist shear without reinforcement. If it is not, then the equation for V_s , the capacity provided by the steel stirrup reinforcement, comes into play. The design approach is to select a shear stirrup size, A_v , and solve for the spacing of the stirrups, s .

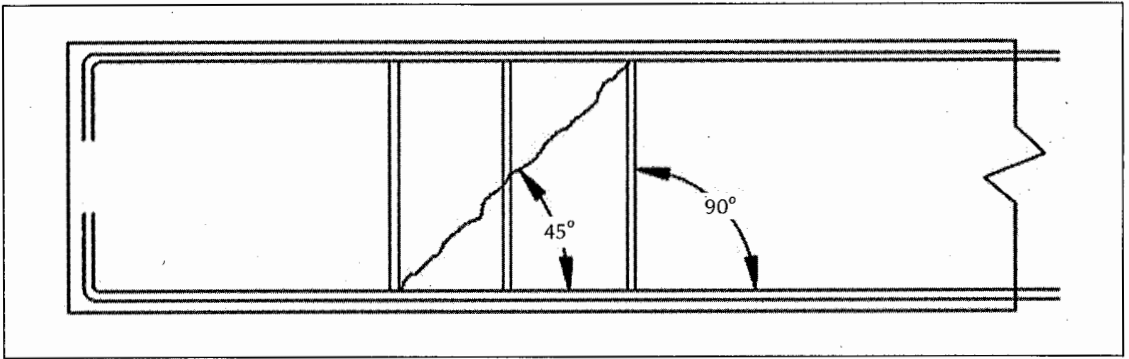


FIGURE 3. Traditional design schematic with simplifying assumptions.

As is evident through this derivation, the traditional model for shear design in concrete assumes that no tensile stress is transferred across the shear cracks and all tensile forces are taken by the tensile steel and vertical stirrups. While these assumptions greatly simplify the process of designing for shear in concrete bridge design, their validity has been challenged in recent years.

Modified Compression Field Theory Derivation

Recent experimentation has determined that the simplifying assumptions presented in the traditional method of concrete shear design are very conservative. The shear crack angle is variable. Also, counter-intuitively, it has been proven possible for tensile stresses to be transferred across these shear cracks, which happens due to the non-homogenous nature of concrete. The aggregate tends to "lock up" as the planes of the crack slip in opposite directions. As long as the axial strain of the member (ϵ_x) is kept to a minimum, the section remains whole and tension is, in fact, transferred across the diagonal shear cracks. This minimal axial strain is obtained by ensuring that the crack size is limited, allowing the aggregate particles to interlock and carry tension. Figure 4 depicts the pre-cracked condition of concrete, with the principal stresses f_1 and f_2 of the stress element equal to each other (a); the idealized concrete beam present in the traditional model of shear design, where θ is equal to 45 and f_1 is non-existent (b); and the idealized concrete beam present in the modified compression field theory, where θ is less

than 45 and f_1 is not equal to 0 (c). Figure 5 shows the resulting schematic of forces and stresses in a cracked beam, according to MCFT.

Summing the vertical forces in Figure 5a yields:

$$\Sigma F_v = V = f_2 b_v d_v \cos(\theta) \sin(\theta) + f_1 b_v d_v \sin(\theta) \cos(\theta) \quad (10)$$

where:

f_1 = the average tensile stress across the concrete and d_v is the effective shear width, taken as the distance between the resultants of the tensile and compressive forces due to flexure.

If v is equal to the average shear stress across concrete, then:

$$v = V / (b_v d_v) \quad (11)$$

$$v b_v d_v = f_2 b_v d_v \cos(\theta) \sin(\theta) + f_1 b_v d_v \sin(\theta) \cos(\theta) = (f_2 + f_1) \sin(\theta) \cos(\theta) \quad (12)$$

Solving for f_2 :

$$f_2 = (v / \sin(\theta) \cos(\theta)) - f_1 \quad (13)$$

If the force in the stirrups is considered, summing the vertical forces in Figure 5b yields:

$$\Sigma F_v = A_v f_v = f_2^* s^* b_v^* \sin^2(\theta) - f_1^* s^* b_v^* \cos^2(\theta) \quad (14)$$

By substituting and simplifying, the following relationship can be obtained:

$$V = (A_v f_v d_v / s) \cot(\theta) + f_1 b_v d_v \cot(\theta) \quad (15)$$

This result can be compared to the relationship given by the traditional shear derivation:

$$V = (A_v f_v d_v / s) \cot(\theta) \quad (16)$$

where:

$$\cot(\theta) = \cot(45) = 1$$

The difference between the two theories is the $f_1 b_v d_v \cot(\theta)$ term, which represents the force due to tension across the shear cracks.

Tension can exist in the modified compression field only if slippage across the cracks is limited. Figure 6 illustrates the tensile forces across a crack, where (a) shows a beam web cracked by shear, (b) shows the average stresses between cracks and (c) shows the local stresses at a crack. The local value of shear stress at the crack, v_{ci} , can be determined by:

$$v_{ci} \leq (0.0683 \sqrt{f'_c}) / (0.3 + 24w/a_{max} + 0.63) \quad (17)$$

where:

v_{ci} = the local value of shear stress at the crack,

a_{max} = the maximum aggregate size, and
 w = the crack width.

Reverting to Equation 15, f_1 can be taken to be equal to the average tension stress (see Figures 5 & 6), thus:

$$f_1 = v_{ci} \tan(\theta) \leq (0.0683 \sqrt{f'_c}) \tan(\theta) / (0.3 + 24w/a_{max} + 0.63) \quad (18)$$

AASHTO prefers to express Equation 18 as:

$$f_1 = v_{ci} \tan(\theta) \leq (2.16 * 0.0316 \sqrt{f'_c}) \tan(\theta) / (0.3 + 24w/a_{max} + 0.63) \quad (19)$$

where:

0.0316 = a conversion factor from psi to ksi (for f'_c) to accommodate AASHTO's preference for f'_c in ksi, in contrast to the ACI approach for f'_c to be in psi.

Thus, the expression for the nominal shear

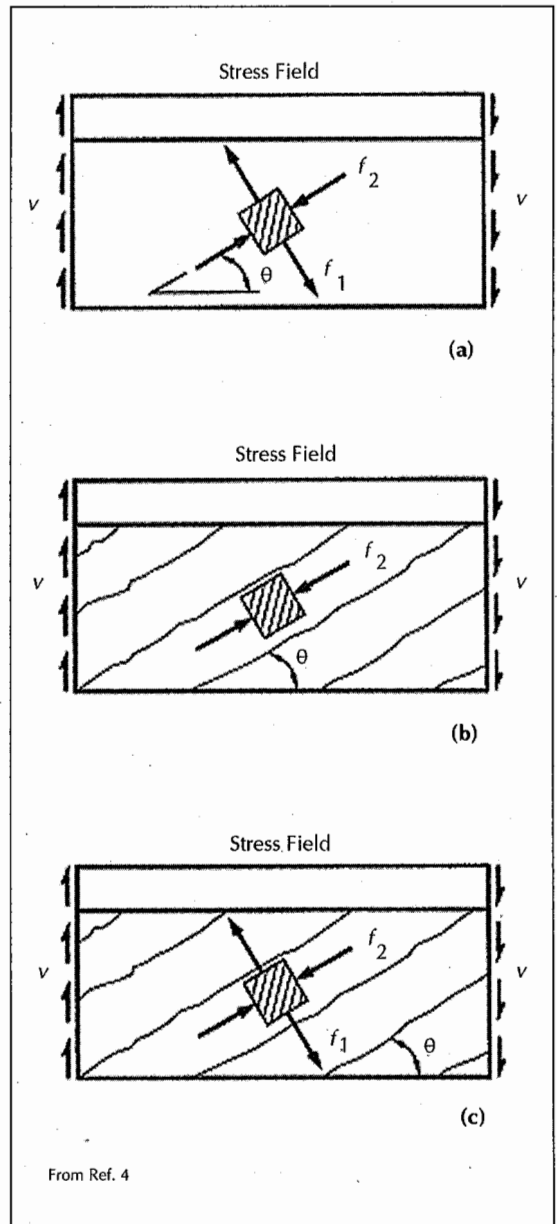
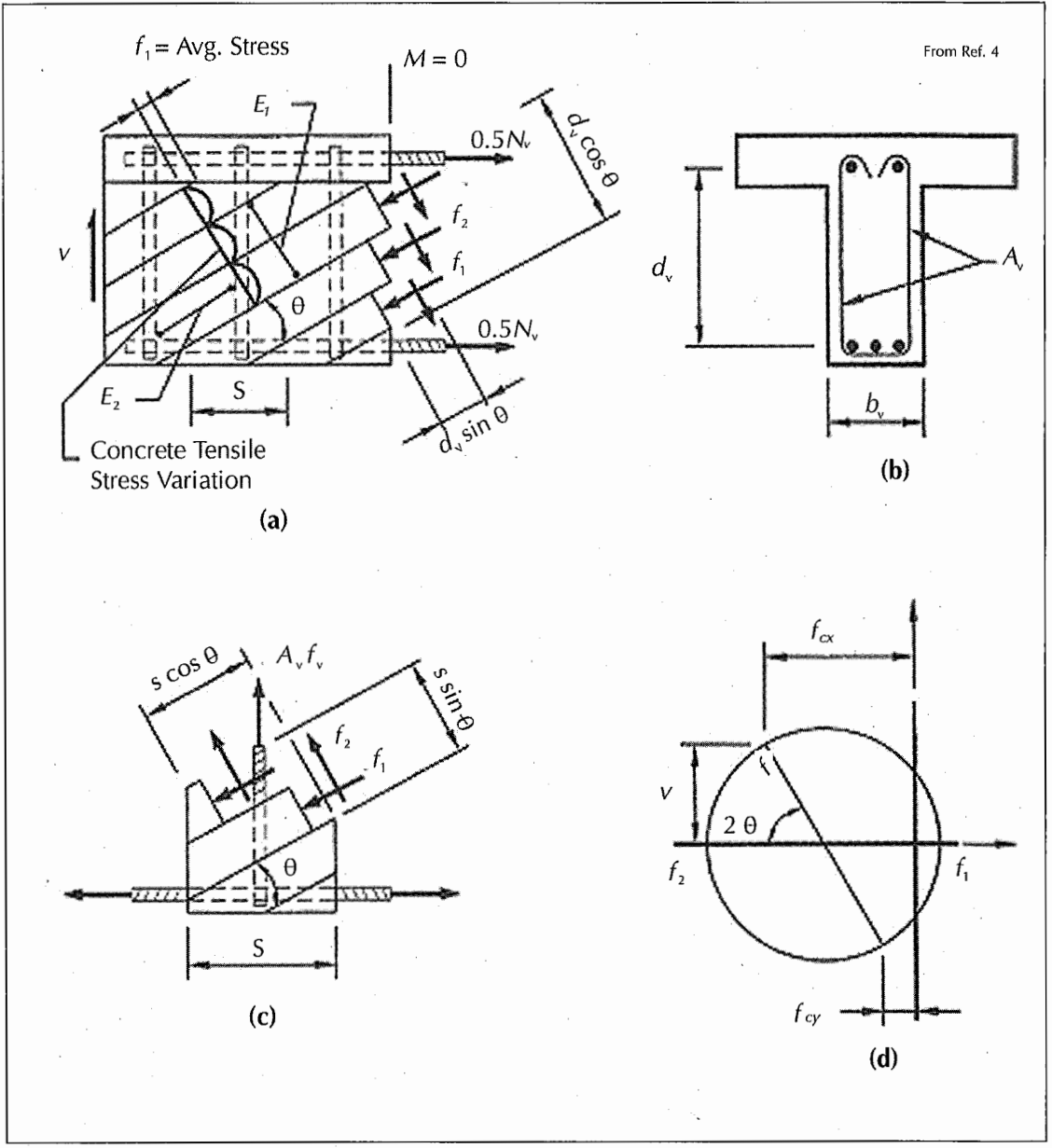


FIGURE 4. Comparison of stress fields in a concrete beam subjected to shear.

strength of a reinforced concrete beam becomes:

$$V = (A_v f_v d_v / s) \cot(\theta) + (2.16 * 0.0316 \sqrt{f'_c}) / [(0.3 + 24w/a_{max} + 0.63) b_v d_v] \quad (20)$$

If a new variable, β , is defined as $2.16 \sqrt{f'_c} / (0.3 + 24w/a_{max} + 0.63)$, Equation 20 simplifies to its final form:



From Ref. 4

FIGURE 5. Schematic of forces and stresses in a concrete beam subjected to shear.

$$V = (A_v f_v d_v / s) \cot(\theta) + 0.0316 \beta \sqrt{f'_c} (b_v d_v) \quad (21)$$

Note that for θ equal to 45 degrees and β equal to 2, this expression becomes the same as the traditional ACI method of shear calculation.

Numerical Comparison of Traditional & MCFT Methods

Values of V_{sr} , the required shear resistance of the added steel stirrups, and s , the required

spacing of stirrups to provide this resistance, were calculated for the example shown in Figure 7 using both the traditional model and the MCFT method. Results of these calculations are summarized in Table 1.

The problem statement assumes the following:

$$V_u = 157 \text{ kip}$$

$$M_u = 220 \text{ kip-ft}$$

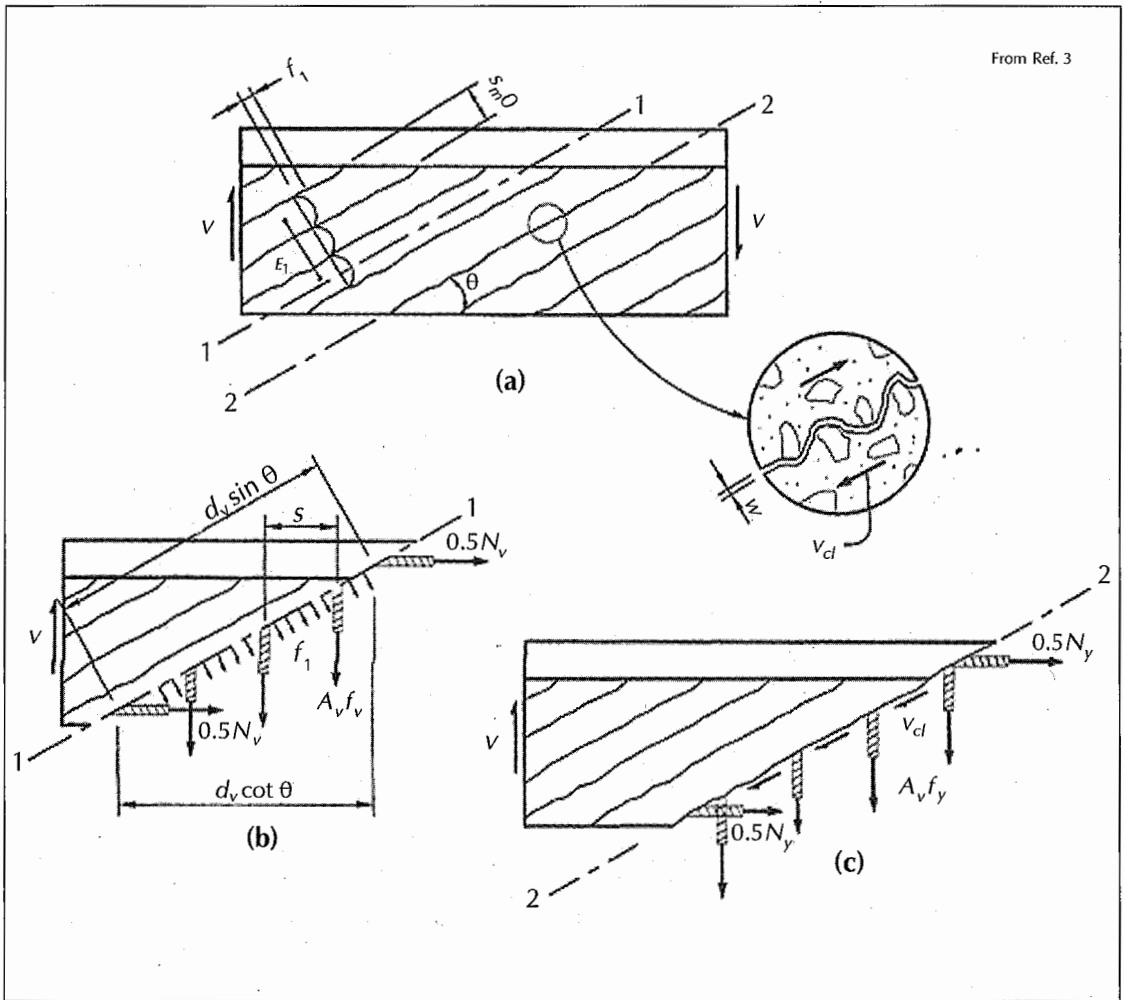


FIGURE 6. Tensile forces across a crack.

$$f'_c = 4,500 \text{ psi}$$

$$f_y = 60,000 \text{ psi}$$

Interestingly, the required shear resistance of the concrete, V_{sr} , is 8.3 percent greater using MCFT. In effect, the modified "cost-saving" method counter-intuitively delivers a concrete beam section that has a lower nominal shear resistance. However, the shear strength of the concrete is not the governing factor in this calculation.

Regarding the required stirrup spacing (the real result of interest), a much more distinct difference between the two methods can be observed. The modified compression field theory model predicts a required stirrup spacing of 7.73 inches, 25.7 percent

greater than the traditional model prediction of 6.15 inches. Although this larger spacing may not seem possible because the modified method requires a larger shear resistance, the treatment of the crack angle (β in the traditional method and θ in the modified method) in each derivation should be recalled. Both calculations have a $\cot(\text{crack angle})$ multiplier at the end of the calculation. However, unlike the traditional method, which assumes a conservative crack angle of 45 degrees for simplification to make $\cot(45)$ equal to 1, MCFT assumes a variable crack angle.

This variable crack angle (always taken to be less than 45 degrees) turns the $\cot(\beta)$ term into a multiplier, rather than a disappearing

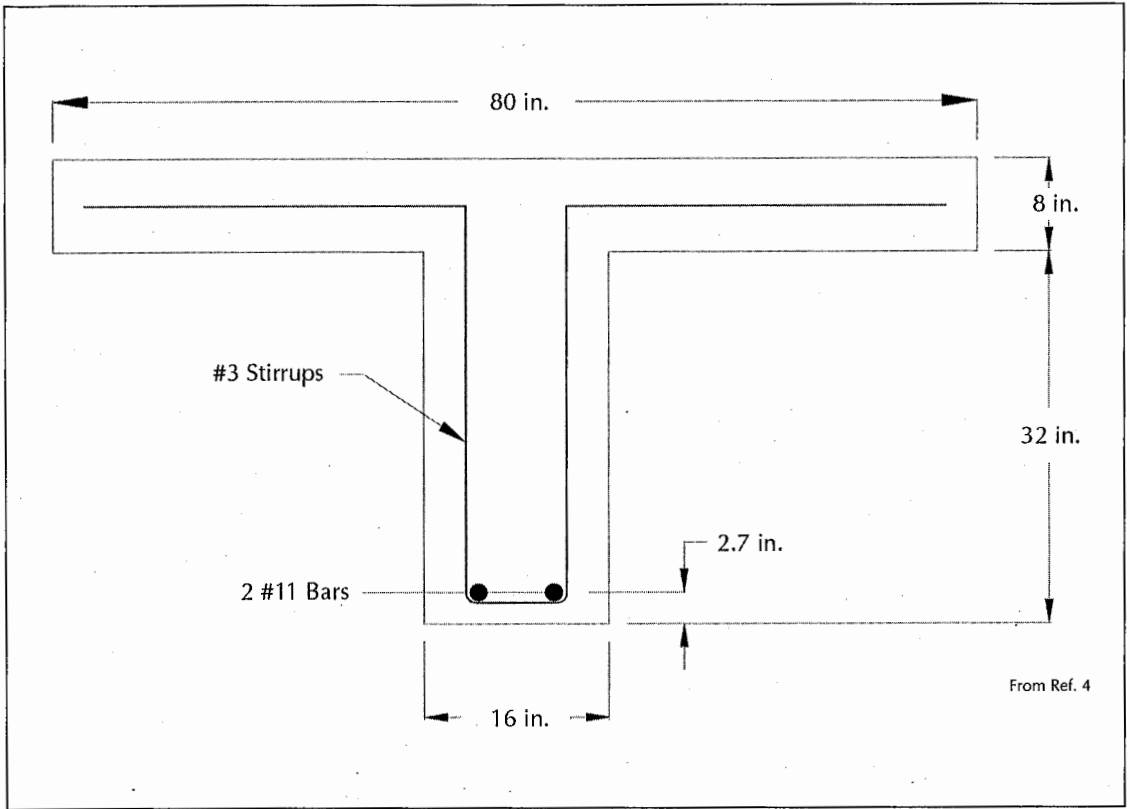


FIGURE 7. Numerical example for comparison of traditional and modified methods.

term. To illustrate this point, the preceding example, in which iteration ultimately predicts the MCFT crack angle, θ , to be 36.4 degrees. The multiplier becomes $\cot(36.4)$, or 1.36. Although MCFT predicts an 8.3 percent decrease in nominal shear resistance, the variable crack angle compensates for this loss with a 36 percent increase in the crack spacing calculation over the traditional method, and a resulting 25.7 percent increase in overall stirrup spacing.

To put this calculation into perspective, using the parameters and dimensions given in the preceding example, by the modified compression field approach a 25-foot beam requires approximately 80 percent of the shear reinforcement steel required by the traditional approach.

Iteration & Automation of MCFT

Application of the traditional method for

TABLE 1.
A Comparison of Numerical Example Results

| | Traditional Model | Modified Compression Field Theory | Percent Difference |
|----------------|-------------------|-----------------------------------|--------------------|
| Required V_s | 79.40 kip | 85.96 kip | 8.3 |
| Required s | 6.15 in | 7.73 in | 25.7 |

analysis and design is relatively straightforward. Unfortunately, as can be inferred by the comparison presented in the previous section, application of the new AASHTO method based on MCFT is not. It requires an iterative process in which different angles of θ are calculated and then tested against a group of assumptions summarized by the new variable, β (not to be confused with the crack angle, β , in the traditional method). These simplifying assumptions, developed by Collins and Mitchell, relate to factors of horizontal strain (or a maximum crack width), which incorporate a maximum aggregate size of 0.75 inches and a maximum crack spacing of 12 inches.⁵

An automated tool was developed to simplify the process of selecting β . This tool, which is programmed into the Visual Basic editor in a Microsoft Excel spreadsheet, requires the user to enter an estimate of θ (crack angle) and then calculates the resulting axial strain, ϵ_x . The spreadsheet then performs several iterations using the table found in AASHTO Figure 5.8.3.4.2-1 to obtain values of β and θ (see Table 2).⁶

The modified shear design process mirrors that of the traditional method from this point on. The difference is that in the traditional method, β is assumed equal to 2 and θ is equal to 45 degrees, whereas the modified method uses a more conservative higher value of β , and a less conservative, lower value of θ . The difference in crack angle overcompensates for the higher value of β , ultimately providing a higher required shear stirrup spacing.

Parametric Study Comparing Different Beam Designs

The required spacing of shear reinforcement obtained from both the traditional design method was compared to the new MCFT design method specified by AASHTO for a variety of beam geometries and distributed loads. Standard beam cross-sections were selected for beam lengths of 30, 40, 50 and 60 feet, and shear reinforcement spacings were calculated using loads of 0, 1,000, 2,000, 3,000, 4,000, 6,000, 7,500 and 10,000 pounds per foot (lbs/ft). The study assumed 60 ksi steel, 4 ksi concrete and #3 bars for shear reinforcement

(this assumption may not always be realistic, but for theoretical considerations, the resulting numbers are comparable).

For loads up to 3,000 to 4,000 lbs/ft (depending on beam size), maximum spacing requirements governed the reinforcement spacing. Because the reinforcement spacing does not depend on the method of design for these cases, but rather AASHTO requirements based on beam geometry, the resulting spacings were omitted from this study.

The results of the remaining cases displayed a significant variation between the traditional and MCFT methods of design in shear bar spacing. The stirrup spacings calculated using the newer, more complex MCFT method of design were an average of 107 percent larger than those calculated with the traditional method of design for those cases not controlled by maximum spacing requirements. In this study, shear spacing calculated at the point of maximum shear along the beam was evaluated. The results using MCFT suggest significant potential savings in shear stirrups due to increased bar spacing. Maximum allowable spacing will probably be found to govern the design in more locations than when using the traditional design method. The potential materials savings make this method attractive from an economic standpoint, assuming the costs associated with engineering efforts are comparable among the two design methods.

Although there is a significant contrast between the reinforcement spacings obtained using the new AASHTO LRFD method compared to the traditional design method, the values for β and θ obtained using the new method did not vary significantly among themselves. Possible values of θ range from 18.1 to 37.3 degrees, yet those in the parametric study ranged from only 26.6 to 32.7 degrees. Similarly, possible values of β range from 1.50 to 6.32, yet those in the study ranged from only 1.86 to 2.94.

Although a more extensive study could be used to obtain a greater variation in results, the relative uniformity of variables in the results of this study suggest that conservative near-average values of β and θ could be substituted into the equation to simplify the itera-

TABLE 2.
Values of θ & β for Sections With Transverse Reinforcement

| $V_u/f'c$ | $\epsilon_x \times 1000$ | | | | | | | | |
|--------------|--------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| | ≤ -0.20 | ≤ -0.10 | ≤ -0.05 | ≤ 0 | ≤ 0.125 | ≤ 0.25 | ≤ 0.50 | ≤ 0.75 | ≤ 1.00 |
| ≤ 0.075 | 22.3 6.32 | 20.4 4.75 | 21.0 4.10 | 21.8 3.75 | 24.3 3.24 | 26.6 2.94 | 30.5 2.59 | 33.7 2.38 | 36.4 2.23 |
| ≤ 0.100 | 18.1 3.79 | 20.4 3.38 | 21.4 3.24 | 22.5 3.14 | 24.9 2.91 | 27.1 2.75 | 30.8 2.50 | 34.0 2.32 | 36.7 2.18 |
| ≤ 0.125 | 19.9 3.18 | 21.9 2.99 | 22.8 2.94 | 23.7 2.87 | 25.9 2.74 | 27.9 2.62 | 31.4 2.42 | 34.4 2.26 | 37.0 2.13 |
| ≤ 0.150 | 21.6 2.88 | 23.3 2.79 | 24.2 2.78 | 25.0 2.72 | 26.9 2.60 | 28.8 2.52 | 32.1 2.36 | 34.9 2.21 | 37.3 2.08 |
| ≤ 0.175 | 23.2 2.73 | 24.7 2.66 | 25.5 2.65 | 26.2 2.60 | 28.0 2.52 | 29.7 2.44 | 32.7 2.28 | 35.2 2.14 | 36.8 1.96 |
| ≤ 0.200 | 24.7 2.63 | 26.1 2.59 | 26.7 2.52 | 27.4 2.51 | 29.0 2.43 | 30.6 2.37 | 32.8 2.14 | 34.5 1.94 | 36.1 1.79 |
| ≤ 0.225 | 26.1 2.53 | 27.3 2.45 | 27.9 2.42 | 28.5 2.40 | 30.0 2.34 | 30.8 2.14 | 32.3 1.86 | 34.0 1.73 | 35.7 1.64 |
| ≤ 0.250 | 27.5 2.39 | 28.6 2.39 | 29.1 2.33 | 29.7 2.33 | 30.6 2.12 | 31.3 1.93 | 32.8 1.70 | 34.3 1.58 | 35.8 1.50 |

From Ref. 6

tion process of the new AASHTO LRFD shear design method.

Discussion of NCHRP Report 549 & Proposed Code Changes

In 2005, the National Cooperative Highway Research Program (NCHRP) released Report 549, entitled *Simplified Shear Design of Structural Concrete Members*.⁷ The report provides an overview of a research program conducted by the NCHRP that attempted to develop practical equations for the design of shear reinforcement, specifically focused on reinforced and prestressed concrete bridge girders. Report 549 provides several recommendations for the improvement of the existing shear reinforcement design method outlined by AASHTO, with the intention that these improvements would be considered for the 2007 AASHTO Bridge Design Specifications.

The NCHRP research began by reviewing the "structure and underlying bases" of sev-

eral well-known methods of calculating shear capacity. These methods included old ACI and AASHTO methods, as well as present and past international methods of calculation. The most significant of these methods turned out to be the Canadian Standards Association (CSA) Code for the Design of Concrete Structures, CSA A23.3-04.⁸ The CSA A23.3-04 design method is based on the same principles as the new AASHTO method, but it is far less cumbersome since it provides simple methods for calculating β and θ . To calculate β for members with A_v less than $A_{v, min}$:

$$\beta = [4.8/(1+1500\epsilon_x)]*[51/(39+s_{xe})] \quad (22)$$

To calculate β for members with A_v greater than or equal to $A_{v, min}$ (note that s_{xe} equals 12 inches):

$$\beta = 4.8/(1+1500\epsilon_x) \quad (23)$$

To calculate θ :

$$\theta = 29 + 7000\epsilon_x \quad (24)$$

After conducting a series of field experiments, the researchers concluded that both the CSA method and the new AASHTO LRFD method of shear reinforcement design were the most accurate of all of methods surveyed, and had only approximately a 10 percent chance of being unconservative.⁷ With the preceding equations in mind, it is easy to see the advantages to the CSA method of design compared to the cumbersome AASHTO iterative method of determining β and θ .

Additionally, state departments of transportation and bridge designers were surveyed regarding the new MCFT method of shear reinforcement design in comparison to the traditional method of design.⁷ Of the findings, the most significant were that, in general, bridge designers had little experience using the new AASHTO LRFD shear design specifications, and that everyone in the profession agreed that the new provisions must be computer-automated if AASHTO is going to require their use. Unfortunately, the implications of both the use and automation of the new method include the engineers' loss of an intuitive sense of the design, and subsequently their comfort in carrying out the required calculations.

Upon completion of this project, the NCHRP researchers produced a series of recommendations to improve the current AASHTO LRFD Bridge Design Specifications.⁷ Among these recommendations was a new method of designing shear reinforcement, which was a modification of the existing method that incorporated the CSA A23.3-04 method of design. These modifications would make the currently required calculation much simpler, and still provide a conservative yet efficient design procedure for reinforced and prestressed concrete bridge girders in shear.

At its 2007 annual meeting, the AASHTO Subcommittee on Bridges and Structures adopted Agenda Item 34 (among others), as a 2008 interim change to the 2007 AASHTO LRFD Bridge Design Specifications.⁹ This item was a result of the NCHRP report, and intro-

duces a more general method of calculating β and θ . Agenda Item 34 presents equations that allow for the direct solution of β , and suggests that θ be taken as 30 degrees in all cases, making the new method non-iterative. These changes, if permanently adopted, would greatly increase the efficiency of the MCFT design method. Although this method is attractive from an economic standpoint, it may take years before the new design method makes intuitive sense to those who use it, and is fully accepted by design engineers.

Conclusions

The rationale behind the new AASHTO LRFD method of designing for shear is well-founded. Its use of MCFT provides a more accurate representation of the true shear strength of reinforced and prestressed concrete beams since the assumptions made in the traditional model of derivation are very conservative. As a result, the new AASHTO method affords the designer reinforcement cost-savings, as well as the opportunity to allow beams to carry more load and span further distances than previously recommended under the traditional design method.

Despite these benefits, however, the disadvantages to the AASHTO method far outweigh its advantages. The new method of design is cumbersome and does not make immediate intuitive sense. As a result, the design process is lengthy and confusing, unlike the more traditional design method. Engineers find it difficult to perform quick mental calculation checks because of the lack of intuition involved in the calculation, and this fact in itself could jeopardize the safety of structures designed under the new provisions.

In response to these difficulties, it is recommended that:

- The next edition of the AASHTO LRFD Bridge Design Specifications should use modified provisions for designing for shear that incorporate the CSA A23.3-04 method of design, as outlined in NCHRP Report 549, and proposed by the AASHTO Subcommittee on Bridges and Structures.

- All concrete codes should work to incorporate these modified design provisions in order to increase the accuracy of design, provide greater simplicity in calculation, reduce costs, and create uniformity across the concrete design field.
- Until these changes are adopted, software that automatically performs the necessary iterations to obtain values for β and θ , similar to the spreadsheet developed herein, should be distributed and utilized by concrete bridge designers to lessen the present complexity of the shear reinforcement design process.

MCFT is a great advancement in the field of concrete design, but the methods of design must strike a balance between utilizing the accuracy of the theory and ensuring efficient design for engineers. If these recommendations are adopted and implemented, both will be obtained and the bridge design process will be more efficient, economical and safe.

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JASON VARNEY is a graduate student at the University of Texas at Austin, where he is pursuing a master's degree in Structural Engineering. He is originally from Belfast,

Maine, and recently completed his B.S. degree in Civil Engineering at Tufts University.

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